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High-Frequency Sum Rules for the Quasi-One-Dimensional Quantum Plasma Dielectric Tensor

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A high-frequency sum-rule expansion is derived for all elements of the spinless quasi-one-dimensional quantum plasma response tensor at $T=0$ K. As in the magnetized classical plasmas, we find that Ω_4^{13} is the only coefficient of ω^{-4} that has no correlational term. Further, we find that the correlations either enhance or reduce the negative quantum dispersion, depending on the direction of propagation. It is also noted that the quantum effect does not exist for the ordinary and the extraordinary modes for perpendicular and parallel propagation, respectively.

1. INTRODUCTION

High-frequency sum-rule expansions of the full response tensor of classical one-component plasmas in the absence and presence of an external magnetic field are known (Kalman and Genga, 1986; Genga, 1988). However, for quantum plasmas with spinless particles the existing results pertain to the absence of an external magnetic field (Genga, submitted). In this case we consider the high-frequency sum-rule expansion to order ω^{-5} for the full response tensor of quasi-one-dimensional quantum nonrelativistic plasmas with spinless particles at $T=0$ K.

In this work we treat an electron plasma in a constant, homogeneous magnetic field quantum mechanically. While treating the magnetic field exactly, a perturbation approach in the photon field is used in deriving the general expressions for the dielectric tensor (Canuto and Ventura, 1972). In laboratory plasmas the magnetic field is of order 10^5 G, which is very small compared to the 10^{15} G found in pulsars. At superstrong magnetic fields such as those probably associated with neutron stars we find that when the Fermi energy of the electrons is lower than the excitation energy

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of the Landau levels, i.e., $p^2/2m \ll \hbar\Omega$, only the lowest $n=0$ level is populated, and the mobility of the electrons is therefore entirely determined by the value of momentum along the z axis, i.e., p_z, thus giving rise to a one-dimensional quantum plasma. Low density, as well as an intense magnetic field, is necessary for this situation to be realized.

Since we are considering an anisotropic system in the presence of an external magnetic field, we calculate the high-frequency sum-rule expansion for the six independent elements of the dielectric tensor of the quasi-one-dimensional quantum plasmas. The method of derivation is reviewed below. In Section 2 we calculate the exact ω^{-2}, ω^{-3}, ω^{-4}, and ω^{-5} sum-rule coefficients for the full response tensor; in Section 3 the long-wavelength limit of the results of Section 2 is considered. Strong coupling effects on the high-frequency modes, i.e., the plasma mode and the high-frequency extraordinary mode for propagation parallel and perpendicular to the magnetic field respectively, are determined in Section 4. The results of Section 2 are obtained by using the same method as the one shown in the Appendix of Genga (in press) for magnetic field-free case.

The total electric current at point $\mathbf{x}_i$ is given by

$$j(\mathbf{x}) = \frac{e}{2}\sum_i [\mathbf{V}_i\,\delta(\mathbf{x}-\mathbf{x}_i) + \delta(\mathbf{x}-\mathbf{x}_i)\mathbf{V}_i] \tag{1}$$

where

$$\mathbf{V}_i = \frac{1}{m}\left[\mathbf{P}_i + \frac{e}{c}\bar{A}^0(\mathbf{x}_i) + \frac{e}{c}\bar{A}(\mathbf{x}_i, t)\right] \tag{2}$$

$\mathbf{x}_i$, $\mathbf{V}_i$, $\mathbf{P}_i$, $\bar{A}^0(\mathbf{x}_i)$ and $\bar{A}(\mathbf{x}_i, t)$ correspond to the position, velocity, momentum, external field vector, and self-consistent field vector of the ith particle, respectively. In Fourier transform language equation (1) becomes

$$\langle J^{\mu}_{\mathbf{k}}(\omega)\rangle = e\langle j^{\mu}_{\mathbf{k}}(\omega)\rangle + \frac{e^2 N}{mc} T^{\mu\nu}_{\mathbf{k}} A_{\mathbf{k}}(\omega) \tag{3}$$

where

$$T^{\mu\nu}_{\mathbf{k}} = 1 - k^{\mu}k^{\nu}/k^2 \tag{4}$$

We replace the Fourier transform of equation (1) by its expectation value to obtain equation (3), since we are only interested in the response function of the electron system. By applying perturbation theory (Genga, in press; Pines and Nozières, 1966), we find that

$$\langle j^{\mu}_{\mathbf{k}}(\omega)\rangle = -\frac{e^2}{c}\sum_{np} \omega^{-1}\langle 0|\Pi^{\mu} k(\tau)|n\rangle\langle|\Pi^{\nu}_{-\mathbf{k}}(0)|0\rangle \times\left[\frac{1}{\omega-\omega_{n0}(p, p+\hbar k/2)+i\eta} - \frac{1}{\omega+\omega_{n0}(p, p-\hbar k/2)+i\eta}\right] A^{\nu}_{\mathbf{k}}(\omega) \tag{5}$$

where

$$\Pi^{\mu}_{\mathbf{k}} = \frac{1}{2m} \sum_i [\Pi^{\mu}_i \exp(i\mathbf{k}\cdot\mathbf{x}_i) + \exp(i\mathbf{k}\cdot\mathbf{x}_i)\,\Pi^{\mu}_i]$$
$$\Pi^{\mu}_i = p^{\mu}_i + \frac{e}{c}\bar{A}^{0\mu}(\mathbf{x}_i) \tag{6}$$

and for the arguments of ω_{n0} as well as the summation over p in equation (5)

$$p = p_z, \qquad k = k_z \tag{7}$$

Then, by combining equations (3) and (5) we obtain

$$\sigma^{\mu\nu}(\mathbf{k}\omega) = \frac{ie^2}{\omega}\left[\chi^{\mu\nu}(k\omega) + \frac{N}{m}T^{\mu\nu}_k\right] \tag{8}$$

where $\chi^{\mu\nu}$ is the current–current response tensor defined as

$$\chi^{\mu\nu}(\mathbf{k}\omega) = \sum_{np} \langle 0|\Pi^{\mu}_k(\tau)|n\rangle\langle n|\Pi^{\nu}_{-k}(0)|0\rangle \times\left[\frac{1}{\omega - \omega_{n0}(p, p+\hbar k/2) + i\eta} - \frac{1}{\omega + \omega_{n0}(p, p-\hbar k/2) + i\eta}\right] \tag{9}$$

Since the polarizability tensor, $\alpha^{\mu\nu}$ and the conductivity tensor $\sigma^{\mu\nu}$ are related, i.e.,

$$\alpha^{\mu\nu}(\mathbf{k}\omega) = i\frac{4\pi e^2}{\omega}\sigma^{\mu\nu}(\mathbf{k}\omega)$$

we find that equation (8) can be expressed as

$$\alpha^{\mu\nu}(\mathbf{k}\omega) = \frac{\omega_p^2}{\omega^2}T^{\mu\nu}_k + \bar{\alpha}^{\mu\nu}(\mathbf{k}\omega) \tag{10}$$

where

$$\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega) = -4\pi e^2\frac{\chi^{\mu\nu}}{\omega^2}(\mathbf{k}\omega) \tag{11}$$

In this work we consider equation (11), since the first term in equation (10) is already in the expansion form. The matrix elements and excitation frequencies that appear in equation (9) are those appropriate for a system of electrons with Coulomb and external magnetic field interactions, but without any transverse self-consistent magnetic field interactions.

2. SUM RULES

The complete modified polarizability tensor $\bar{\alpha}^{\mu\nu}(\mathbf{k})$ is expressible in terms of the corresponding "external" quantities $\hat{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ as (Genga, in press)

$$\bar{\alpha}(\mathbf{k}\omega) = \hat{\alpha}(\Delta - \hat{\alpha})\Delta \tag{12}$$

where

$$\Delta = 1 - n^2 T, \qquad n = kc/\omega, \qquad T = 1 - k \cdot k/k^2 \tag{13}$$

This is because $\hat{\alpha}^{\mu\nu}(k\omega)$ possesses the well-known high-frequency sum-rule expansion (Genga, in press)

$$\hat{\alpha}^{H'\mu\nu}(k\omega) = -\sum_{\substack{l=1\\ l=\text{odd}}} \frac{\hat{\Omega}^{\mu\nu}_{l+1}(k)}{\omega^{l+1}} \tag{14}$$

$$\hat{\alpha}^{H''\mu\nu}(\mathbf{k}\omega) = -\sum_{\substack{l=2\\ l=\text{even}}} \frac{\hat{\Omega}^{\mu\nu}_{l+1}(\mathbf{k})}{\omega^{l+1}} \tag{15}$$

as $\hat{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ in the classical case (Kalman and Genga, 1986; Genga, 1988). The superscript H stands for "Hermitian part of," and prime and double prime denote "real part of" and "imaginary part of," respectively. As in the magnetic field-free case (Genga, in press) the $\hat{\Omega}^{\mu\nu}$ coefficients are calculated from the relation

$$\begin{aligned}\hat{\Omega}^{\mu\nu}_{l+1}(\mathbf{k}) = 4\pi e^2 \sum_{np} \{&[\omega_{n0}(p, p - \hbar k/2)]^{l-2} \langle 0|\Pi^{\mu}_{\mathbf{k}}|n\rangle\langle n|\Pi^{\nu}_{-\mathbf{k}}|0\rangle \\ &-[-\omega_{n0}(p, p + \hbar k/2)]^{l-2} \langle 0|\Pi^{\nu}_{-k}|n\rangle\langle n|\Pi^{\mu}_{k}|0\rangle\}_{t=0}\end{aligned} \tag{16}$$

It is also known (Kalman and Genga, 1986; Genga, 1988, and in press) that the high-frequency expansion of $\bar{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ becomes similar to that of $\hat{\alpha}^{\mu\nu}(\mathbf{k}\omega)$ as given by equations (14) and (15), with $\Omega^{\mu\nu}_{l+1}(\mathbf{k})$ replacing the corresponding $\hat{\Omega}^{\mu\nu}_{l+1}(\mathbf{k})$. The relationships between the two sets of coefficients up to $l = 4$ are

$$\begin{aligned}\bar{\Omega}^{\mu\nu}_2 &= \hat{\Omega}^{\mu\nu}_2 \\ \bar{\Omega}^{\mu\nu}_3 &= \Omega^{\mu\nu}_3 \\ \bar{\Omega}^{\mu\nu}_4 &= \hat{\Omega}^{\mu\nu}_4 - \hat{\Omega}^{\mu\alpha}_2\hat{\Omega}^{\alpha\nu}_2 \\ \bar{\Omega}^{\mu\nu}_5 &= \hat{\Omega}^{\mu\nu}_5 - \hat{\Omega}^{\nu\alpha}_2\hat{\Omega}^{\alpha\nu}_3 - \hat{\Omega}^{\mu\alpha}_3\hat{\Omega}^{\alpha\nu}_2\end{aligned} \tag{17}$$

The Hamiltonian of the system that satisfies equation (16) is given by

$$H = \sum_i \frac{\Pi_i^2}{2m} + \tfrac{1}{2}\sum_{i\neq j} V(\mathbf{x}_i - \mathbf{x}_j) \tag{18}$$

where $V(\mathbf{x}_i - \mathbf{x}_j)$ is the interaction potential between a pair of particles and is independent of velocity.

We now turn the calculation of the frequency moments (up to $l = 4$). Since we are considering an anisotropic system in the presence of an external magnetic field, $\bar{\alpha}^{\mu\nu}$ is nondiagonal (Genga, 1988). In this case both even and odd moments of $\Omega_{l+1}^{\mu\nu}$ exist. The real diagonal and off-diagonal elements satisfy the symmetric condition

$$\bar{\Omega}_{l+1}^{\mu\nu}(k) = \bar{\Omega}_{l+1}^{\nu\mu}(k) \tag{19}$$

and the imaginary off-diagonal elements satisfy the antisymmetric condition

$$\bar{\Omega}_{l+1}^{\mu\nu}(k) = -\bar{\Omega}_{l+1}^{\nu\mu}(k) \tag{20}$$

as in classical plasmas (Genga, 1988).

The first moment leads to

$$\begin{aligned}\hat{\bar{\Omega}}_2^{\mu\nu}(\mathbf{k}) &= 4\pi e^2 \sum_{np} \left[\frac{\langle 0|\Pi_k^\mu|n\rangle\langle n|\Pi_{-k}^\nu|0\rangle}{\omega_{n0}(p, p+\hbar/2)} + \frac{\langle 0|\Pi_{-k}^\nu|n\rangle\langle n|\Pi_k^\mu|0\rangle}{\omega_{n0}(p, p-\hbar k/2)} \right] \\ &= \omega_p^2 L^{\mu\nu}\end{aligned} \tag{21}$$

The second moment yields

$$\begin{aligned}\hat{\bar{\Omega}}_3^{\mu\nu}(\mathbf{k}) &= 4\pi e^2 \sum (\langle 0|\Pi_k^\mu|n\rangle\langle n|\Pi_{-k}^\nu|0\rangle - \langle 0|\Pi_{-k}^\nu|n\rangle\langle n|\Pi_k^\mu|0\rangle) \\ &= \tfrac{1}{2}(\langle 0|[\Pi_\mathbf{k}^\mu, \Pi_\mathbf{k}^\nu]|0\rangle\langle 0|[\Pi_{-k}^\nu, \Pi_k^\mu]|0\rangle) \\ &= \frac{i\omega_p^2}{m}\frac{eB_\eta^0}{c}\varepsilon^{\mu\eta\nu}\end{aligned} \tag{22}$$

The third moment is given by

$$\begin{aligned}\hat{\bar{\Omega}}_4^{\mu\nu}(\mathbf{k}) &= 4\pi e^2 \sum_{np} \left[\omega_{n0}\left(p, p-\frac{\hbar k}{2}\right) \langle 0|\Pi_k^\mu|n\rangle\langle n|\Pi_k^\nu|0\rangle \right. \\ &\quad \left. + \omega_{n0}\left(p, p+\frac{\hbar k}{2}\right) \langle 0|\Pi_{-k}^\nu|n\rangle\langle n|\Pi_k^\mu|0\rangle \right]_{t=0} \\ &= 2\pi e^2 \langle 0|[[\Pi_k^\mu, H], \Pi_{-k}^\nu] + [[\Pi_{-k}^\nu, H], \Pi_k^\mu]|0\rangle \\ &= -\frac{\omega_p^2 eB_\eta^0}{2mc} k^\alpha \langle 0|\varepsilon^{\mu\eta\nu}\frac{\partial}{\partial x^\alpha} \\ &\quad + \varepsilon^{\mu\eta\alpha}\frac{\partial}{\partial x^\nu} + \varepsilon^{\alpha\eta\nu}\frac{\partial}{\partial x^\mu} + i\varepsilon^{\mu\eta\nu}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{2mc}x^\beta|0\rangle \\ &\quad - \frac{\omega_p^2 eB_\eta^0}{4mc} k^\mu \varepsilon^{\alpha\eta\nu}\langle 0|\frac{\partial}{\partial x^\alpha} + \varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{2mc}x^\beta|0\rangle\end{aligned}$$

$$-\frac{\omega_p^2 eB_\eta^0}{4mc} k^\nu \varepsilon^{\mu\eta\alpha}\langle 0|\frac{\partial}{\partial x^\alpha}+\frac{\varepsilon^{\alpha\eta\beta}eB_\eta^0}{2mc}x^\beta|0\rangle$$

$$-\frac{\omega_p^2 eB_\eta^0}{2mc} k^\alpha k^\mu\langle 0|2mc(B_\eta^0)^{-1}\frac{e^{-1}\partial^2}{\partial x^\alpha\,\partial x^\nu}+i\varepsilon^{\alpha\eta\beta}x^\beta\frac{\partial}{\partial x^\nu}-i\varepsilon^{\alpha\eta\nu}x^\alpha\frac{\partial}{\partial x^\alpha}$$

$$+\varepsilon^{\alpha\eta\mu}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{2mc}(x^\beta)^2|0\rangle-\frac{\omega_p^2 eB_\eta^0}{2mc}k^\alpha k^\nu\langle 0|2mc(B_\eta^0)^{-1}\frac{e^{-1}\partial^2}{\partial x^\alpha\,\partial x^\mu}$$

$$-i\varepsilon^{\alpha\eta\mu}x^\alpha\frac{\partial}{\partial x^\alpha}+i\varepsilon^{\alpha\eta\beta}x^\beta\frac{\partial}{\partial x^\mu}+\varepsilon^{\alpha\eta\mu}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{2mc}(x^\beta)^2|0\rangle$$

$$-\frac{\omega_p^2 eB_\eta^0}{2mc}k^\alpha k^\alpha\langle 0|2mc(B_\eta^0)^{-1}\frac{e^{-1}\partial^2}{\partial x^\mu\,\partial x^\nu}-\varepsilon^{\alpha\eta\nu}x^\alpha\frac{\partial}{\partial x^\mu}$$

$$-i\varepsilon^{\alpha\eta\mu}x^\alpha\frac{\partial}{\partial x^\nu}-\varepsilon^{\mu\eta\alpha}\varepsilon^{\beta\eta\nu}\frac{eB_\eta^0}{2mc}x^\alpha x^\beta|0\rangle+\omega_p^4\langle 0|L_k^{\mu\nu}$$

$$+\frac{1}{N}\sum_q L_q^{\mu\nu}(S_{\mathbf{k}-\mathbf{q}}-S_{\mathbf{q}})|0\rangle \tag{23}$$

The fourth moment yields

$$\hat{\hat{\Omega}}_5^{\mu\nu}(\mathbf{k})=4\pi e^2\sum_{np}\left\{\left[\omega_{n0}\left(p,p-\frac{\hbar k}{2}\right)\right]^2\langle 0|\Pi_k^\mu|n\rangle\langle n|\Pi_{-k}^\nu|0\rangle\right.$$

$$\left.-\left[-\omega_{n0}\left(p,p+\frac{\hbar k}{2}\right)\right]^2\langle 0|\Pi_{-k}^\nu|n\rangle\langle n|\Pi_k^\mu|0\rangle\right\}_{t=0}$$

$$=2\pi e^2\langle 0|[[[\Pi_k^\mu,H],H],\Pi_{-k}^\nu]-[[[\Pi_{-k}^\nu,H],H],\Pi_k^\mu]|0\rangle$$

$$=\frac{\omega_p^2 eB_\eta^0}{4mc}k^\alpha\langle 0|\varepsilon^{\mu\eta\alpha}\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{e^2(B_\eta^0)^2}{4m^2c^2}x^\beta$$

$$+\tfrac{7}{4}\varepsilon^{\alpha\eta\mu}\varepsilon^{\nu\eta\beta}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{mc}x^\beta\frac{\partial}{\partial x^\alpha}+\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{e^2(B_\eta^0)^2}{8m^2c^2}(x^\beta)^2\frac{\partial}{\partial x^\mu}$$

$$+\varepsilon^{\mu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{e^2(B_\eta^0)^2(x^\beta)^2}{8m^2c^2}\frac{\partial}{\partial x^\nu}+i\varepsilon^{\mu\eta\alpha}\varepsilon^{\nu\eta\beta}\varepsilon^{\alpha\eta\beta}e^3\frac{(B_\eta^0)^3(x^\beta)^3}{16m^3c^3}$$

$$+i6\varepsilon^{\mu\eta\alpha}\varepsilon^{\nu\eta\alpha}\frac{eB_\eta^0}{mc}\frac{\partial}{\partial x^\alpha}|0\rangle+\frac{\omega_p^2 eB_\eta^0}{4mc}k^\alpha k^\mu\langle 0|i\tfrac{7}{4}\varepsilon^{\alpha\mu\nu}\frac{\partial^2}{\partial x^\alpha\,\partial x^\alpha}$$

$$+\tfrac{17}{8}\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{mc}x^\beta\frac{\partial}{\partial x^\alpha}+i\tfrac{7}{4}\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{e^2(B_\eta^0)^2}{m^2c^2}(x^\beta)^2|0\rangle$$

$$+\frac{\omega_p^2 eB_\eta^0}{4mc}k^\alpha k^\nu\langle 0|i\tfrac{7}{4}\varepsilon^{\alpha\eta\mu}\frac{\partial^2}{\partial x^\alpha\,\partial x^\alpha}+\tfrac{17}{8}\varepsilon^{\mu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{mc}x^\beta\frac{\partial}{\partial x^\alpha}$$

$$+i\tfrac{7}{4}\varepsilon^{\mu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{e^2(B_\eta^0)^2}{m^2c^2}(x^\beta)^2|0\rangle+\frac{\omega_p^2 eB_\eta^0}{4mc}k^\alpha k^\alpha\langle 0|i6\varepsilon^{\alpha\eta\mu}\frac{\partial^2}{\partial x^\alpha\,\partial x^\nu}$$

$$+i6\varepsilon^{\nu\eta\alpha}\frac{\partial^2}{\partial x^\alpha\,\partial x^\mu}+i\tfrac{3}{2}\varepsilon^{\nu\eta\mu}\frac{\partial^2}{\partial x^\alpha\,\partial x^\alpha}+\tfrac{3}{2}\varepsilon^{\mu\eta\alpha}\varepsilon^{\nu\eta\alpha}\frac{eB_\eta^0}{mc}$$

$$+3\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{mc}x^\beta\frac{\partial}{\partial x^\nu}+3\varepsilon^{\nu\eta\alpha}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{mc}x^\beta\frac{\partial}{\partial x^\mu}$$

$$+i\tfrac{15}{4}\varepsilon^{\mu\eta\alpha}\varepsilon^{\nu\eta\beta}\varepsilon^{\alpha\eta\beta}\frac{e^2(B_\eta^0)^2}{m^2c^2}(x^\beta)^2+i\varepsilon^{\mu\eta\nu}\varepsilon^{\alpha\eta\beta}\frac{eB_\eta^0}{mc}x^\beta|0\rangle$$

$$+i\frac{\omega_p^4 eB_\eta^0}{2mc}\langle 0|L_k^{\mu\nu}+\frac{1}{n}\sum(\varepsilon^{\mu\eta\alpha}L_q^{\alpha\nu}+\varepsilon^{\alpha\eta\nu}L_q^{\alpha\mu})(S_{\mathbf{k}-\mathbf{q}}-S_{\mathbf{k}})|0\rangle \tag{24}$$

where

$$L_{\mathbf{k}}^{\mu\nu}=k^\mu k^\nu/k^2 \tag{25}$$

To obtain an explicit expression for $\bar{\Omega}_{l+1}^{\mu\nu}(k)$, we choose the k-system, in which

$$\begin{aligned}\mathbf{k}&=(0,0,k)\\ B_x^0&=B_i^0=B^0\sin\theta\\ B_y^0&=B_2^0=0\\ B_Z^0&=B_3^0=B^0\cos\theta\end{aligned} \tag{26}$$

and

$$\begin{aligned}q_x&=q^1=q\sin\theta\cos\theta\\ q_y&=q^2=q\sin\theta\sin\phi\\ q_z&=q^3=q\cos\theta\end{aligned} \tag{27}$$

In order to obtain the components of the external magnetic field as given in equation (26), one has (Landau gauge) $A^0=\frac{1}{2}(0, B_z^0x-zB_x^0, 0)$.

3. LONG-WAVELENGTH LIMIT

In the long-wavelength $(k\to 0)$ limit, we find from equations (22)-(24) that the elements of the frequency moments are given by

$$\hat{\bar{\Omega}}_2^{11}(\mathbf{k})=\hat{\bar{\Omega}}_2^{22}(\mathbf{k})=0$$

$$\hat{\bar{\Omega}}_2^{33}(\mathbf{k})=\omega_p^2$$

$$\hat{\bar{\Omega}}_3^{12}(\mathbf{k})=\Omega_3^{21}(\mathbf{k})=i\omega_p^2\Omega\cos\theta$$

$$\hat{\Omega}_3^{23}(\mathbf{k}) = \Omega_3^{32}(\mathbf{k}) = i\omega_p^2\Omega \sin\theta$$

$$\hat{\Omega}_4^{11}(\mathbf{k}) = -\tfrac{2}{15}\frac{\omega_p^2}{m} E_{\text{corr}} k^2$$

$$\hat{\Omega}_4^{13}(\mathbf{k}) = 0 \tag{28}$$

$$\hat{\Omega}_4^{22}(\mathbf{k}) = -\tfrac{2}{15}\frac{\omega_p^2}{m} E_{\text{corr}} k^2$$

$$\hat{\Omega}_4^{33}(\mathbf{k}) = \omega_p^4 - \frac{\omega_p^2}{m}\left(\frac{3p_F^{02}}{m} - \tfrac{4}{15}E_{\text{corr}}\right)k^2$$

$$\hat{\Omega}_5^{12}(\mathbf{k}) = -\hat{\Omega}_5^{21}(\mathbf{k}) = i\omega_p^2\Omega\left[-\left(\frac{3p_F^{02}}{m} + \tfrac{16}{15}E_{\text{corr}}\right)\frac{k^2}{8m}\right]\cos\theta$$

$$\hat{\Omega}_5^{23}(\mathbf{k}) = -\hat{\Omega}_5^{32}(\mathbf{k}) = i\omega_p^2\Omega\left[-\left(\frac{15p_F^{02}}{m} - \tfrac{24}{15}E_{\text{corr}}\right)\frac{k^2}{8m}\right]\sin\theta$$

$|0\rangle$ can easily be shown to be of the form

$$|0\rangle = (2\pi)^{-1/2}\lambda^{-1}e^{(y-y_0)^2}/4\lambda^2 + ip_z/\hbar \tag{29}$$

where

$$\lambda = -\hbar/m\Omega$$

$$y_0 = -\frac{2cp_y}{e}$$

$$y = B_z x - B_x z = -\frac{cp_y}{e} \tag{30}$$

$$\Omega = -\frac{eB^0}{mc}$$

and the last term is the the electron cyclotron frequency. From equation (28) we find that the correlational terms of $\hat{\Omega}_4^{11}$, $\hat{\Omega}_4^{22}$, $\hat{\Omega}_4^{33}$, $\hat{\Omega}_5^{12}$, and $\hat{\Omega}_5^{23}$ are of the same order as their corresponding ones in the classical case (Genga, 1988). E_{corr} is the (negative) correlation energy per particle and p_F^0 is the lowest Landau Level Fermi momentum.

4. STRONG COUPLING EFFECTS ON PLASMA DISPERSION

In this section correlational effects on the undamped high-frequency, quasi-one-dimensional plasma waves, in an external magnetic field, are determined by using high-frequency sum rules (HFSR-s). We limit our problem to quantum nonrelativistic plasmas with spinless particles at $T=0$ K. Although the method is exact, it is not very reliable for the calculation of the dispersion relation (Genga, 1988). The high-frequency modes of interest are the "ordinary" and the "extraordinary" modes; the extraordinary mode under consideration is the one with cutoff frequency $\omega_2=\frac{1}{2}\Omega[1+(1+4\omega_p^2/\Omega^2)^{1/2}]$. All the modes propagating along and across the external magnetic field are considered. We use a coordinate system where $\mathbf{k}=(0,0,k)$ and B^0 is in the $\mathbf{x}$–$\mathbf{z}$ plane, i.e., k-system.

We study the behavior of the system by applying a small perturbation to the dispersion relations (Genga, 1988). As a result of this a frequency shift due to correlations occurs. The frequency shift due to correlations is of order k^2, and thus is small as $k\to 0$ and is equal to the order of the frequency shift due to quantum effects.

4.1. Propagation Parallel to Magnetic Field

In this case only the longitudinal and extraordinary modes exist (Genga, 1988). The longitudinal mode oscillates at the plasma frequency.

4.1.1. Longitudinal Mode

The dispersion relation

$$\varepsilon_{33}(\mathbf{k}\omega)=1+\alpha_{33}(\mathbf{k}\omega)=0 \tag{31}$$

determines the behavior of longitudinal plasmons. After a small perturbation is applied to the dispersion relation the plasmon frequency becomes

$$\omega^2(\mathbf{k})=\omega_p^2-\frac{1}{m}\left[3\,\frac{(P_F^{(0)})^2}{m}-\tfrac{4}{15}E_{\text{corr}}\right]k^2 \tag{32}$$

The correlations are seen to increase the negative quantum dispersion for finite k.

4.1.2. Extraordinary Mode

The dispersion relation that determines the behavior of the extraordinary mode is

$$(\varepsilon_{11}(\mathbf{k}\omega)-n^2)^2-\varepsilon_{12}^2(k\omega)=0 \tag{33}$$

The ensuing frequency can be written as

$$\omega^2(\mathbf{k})=\omega_2^2\left[1+\left(\frac{c^2}{\omega_p^2}-\frac{2}{15m}\frac{E_{\text{corr}}}{\omega_2^2}\right)k^2\right] \tag{34}$$

In this case we see that the correlations enhance the positive refractive dispersion for finite k. It is also noted that the quantum effect on the dispersion does not exist.

4.2. Propagation Perpendicular to Magnetic Field

Unlike the case of propagation along the magnetic field, here we have a pure transverse mode, called the "ordinary mode," and a coupled transverse-longitudinal one, called the "extraordinary mode." The dispersion relation for the ordinary mode is

$$\varepsilon_{11}(k\omega) - n^2 = 0 \tag{35}$$

and that for the extraordinary mode is

$$[\varepsilon_{22}(\omega) - n^2]\varepsilon_{33}(\mathbf{k}\omega) - \varepsilon_{23}^2(\mathbf{k}\omega) = 0 \tag{36}$$

4.2.1. Ordinary Mode

After applying a small perturbation to the dispersion relation, we find that the ensuing frequency becomes

$$\omega^2(k) = \omega_p^2 + \left(c^2 - \frac{2}{15m} E_{\text{corr}}\right) k^2 \tag{37}$$

The effect of correlations is seen to enhance the positive refractive dispersion. We also note that there is no quantum effect on the dispersion.

4.2.2. Extraordinary Mode

In this case the expression for the frequency is

$$\omega^2(\mathbf{k}) = \omega_2^2 \left\{1 + \left[\frac{c^2}{2\omega_p^2} - \frac{1}{m\omega_2^2}(3\varepsilon_F - \tfrac{1}{15}E_{\text{corr}})k^2\right]\right\} \tag{38}$$

where $\varepsilon_F = (P_F^{(0)})^2/2m$ is the Fermi energy per particle. We see that correlations increase the negative quantum dispersion for finite k. The total quantum and correlation effects reduce the positive refractive dispersion for finite k, unlike the case of parallel propagation.

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